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**Mathematics: analysis and approaches**  
**Standard level**  
**Paper 2**

Friday 7 May 2021 (morning)

Candidate session number

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1 hour 30 minutes

**Instructions to candidates**

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.



Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

### Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

At a café, the waiting time between ordering and receiving a cup of coffee is dependent upon the number of customers who have already ordered their coffee and are waiting to receive it.

Sarah, a regular customer, visited the café on five consecutive days. The following table shows the number of customers,  $x$ , ahead of Sarah who have already ordered and are waiting to receive their coffee and Sarah's waiting time,  $y$  minutes.

Number of customers ( $x$ )	3	9	11	10	5
Sarah's waiting time ( $y$ )	6	10	12	11	6

The relationship between  $x$  and  $y$  can be modelled by the regression line of  $y$  on  $x$  with equation  $y = ax + b$ .

- (a) (i) Find the value of  $a$  and the value of  $b$ .  
(ii) Write down the value of Pearson's product-moment correlation coefficient,  $r$ . [3]
- (b) Interpret, in context, the value of  $a$  found in part (a)(i). [1]

On another day, Sarah visits the café to order a coffee. Seven customers have already ordered their coffee and are waiting to receive it.

- (c) Use the result from part (a)(i) to estimate Sarah's waiting time to receive her coffee. [2]

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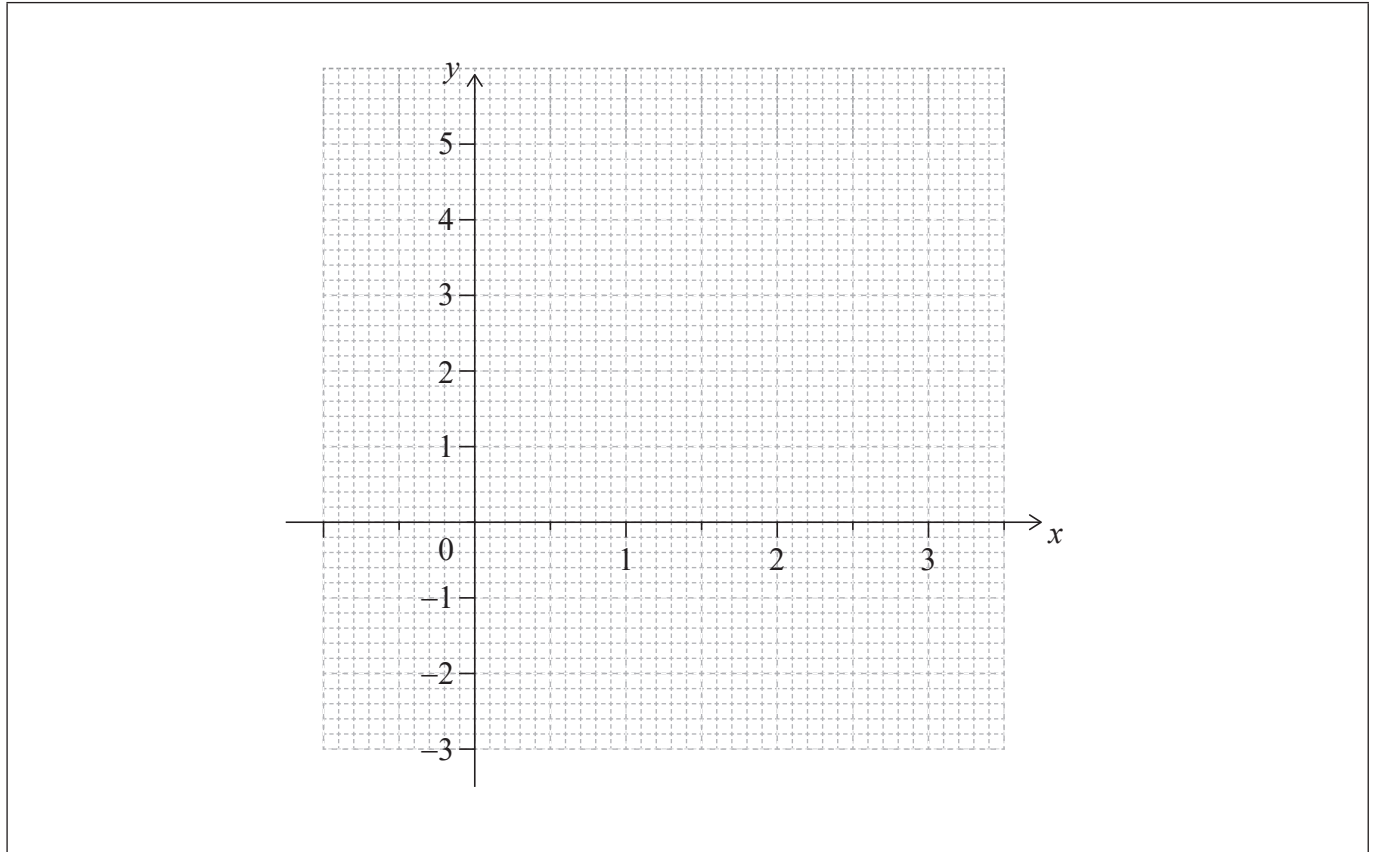
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2. [Maximum mark: 5]

Let  $f(x) = 3x - 4^{0.15x^2}$  for  $0 \leq x \leq 3$ .

(a) Sketch the graph of  $f$  on the grid below. [3]



(b) Find the value of  $x$  for which  $f'(x) = 0$ . [2]

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3. [Maximum mark: 5]

An arithmetic sequence has first term 60 and common difference  $-2.5$ .

(a) Given that the  $k$ th term of the sequence is zero, find the value of  $k$ . [2]

Let  $S_n$  denote the sum of the first  $n$  terms of the sequence.

(b) Find the maximum value of  $S_n$ . [3]

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4. [Maximum mark: 8]

At a school, 70% of the students play a sport and 20% of the students are involved in theatre. 18% of the students do neither activity.

A student is selected at random.

(a) Find the probability that the student plays a sport and is involved in theatre. [2]

(b) Find the probability that the student is involved in theatre, but does not play a sport. [2]

At the school 48% of the students are girls, and 25% of the girls are involved in theatre.

A student is selected at random. Let  $G$  be the event “the student is a girl” and let  $T$  be the event “the student is involved in theatre”.

(c) Find  $P(G \cap T)$ . [2]

(d) Determine if the events  $G$  and  $T$  are independent. Justify your answer. [2]

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5. [Maximum mark: 6]

The functions  $f$  and  $g$  are defined for  $x \in \mathbb{R}$  by  $f(x) = 6x^2 - 12x + 1$  and  $g(x) = -x + c$ , where  $c \in \mathbb{R}$ .

(a) Find the range of  $f$ . [2]

(b) Given that  $(g \circ f)(x) \leq 0$  for all  $x \in \mathbb{R}$ , determine the set of possible values for  $c$ . [4]

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6. [Maximum mark: 7]

All living plants contain an isotope of carbon called carbon-14. When a plant dies, the isotope decays so that the amount of carbon-14 present in the remains of the plant decreases. The time since the death of a plant can be determined by measuring the amount of carbon-14 still present in the remains.

The amount,  $A$ , of carbon-14 present in a plant  $t$  years after its death can be modelled by  $A = A_0 e^{-kt}$  where  $t \geq 0$  and  $A_0, k$  are positive constants.

At the time of death, a plant is defined to have 100 units of carbon-14.

(a) Show that  $A_0 = 100$ . [1]

The time taken for half the original amount of carbon-14 to decay is known to be 5730 years.

(b) Show that  $k = \frac{\ln 2}{5730}$ . [3]

(c) Find, correct to the nearest 10 years, the time taken after the plant's death for 25% of the carbon-14 to decay. [3]

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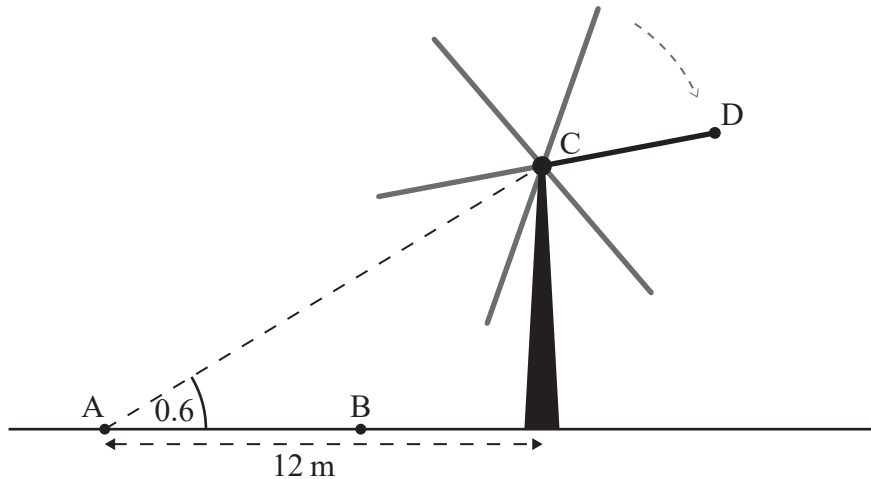
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### Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 13]

The six blades of a windmill rotate around a centre point C. Points A and B and the base of the windmill are on level ground, as shown in the following diagram.



From point A the angle of elevation of point C is 0.6 radians.

- (a) Given that point A is 12 metres from the base of the windmill, find the height of point C above the ground. [2]

An observer walks 7 metres from point A to point B.

- (b) Find the angle of elevation of point C from point B. [2]

The observer keeps walking until he is standing directly under point C. The observer has a height of 1.8 metres, and as the blades of the windmill rotate, the end of each blade passes 2.5 metres over his head.

- (c) Find the length of each blade of the windmill. [2]

One of the blades is painted a different colour than the others. The end of this blade is labelled point D. The height  $h$ , in metres, of point D above the ground can be modelled by the function  $h(t) = p \cos\left(\frac{3\pi}{10}t\right) + q$ , where  $t$  is in seconds and  $p, q \in \mathbb{R}$ . When  $t = 0$ , point D is at its maximum height.

- (d) Find the value of  $p$  and the value of  $q$ . [4]

If the observer stands directly under point C for one minute, point D will pass over his head  $n$  times.

- (e) Find the value of  $n$ . [3]



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8. [Maximum mark: 15]

The flight times,  $T$  minutes, between two cities can be modelled by a normal distribution with a mean of 75 minutes and a standard deviation of  $\sigma$  minutes.

- (a) Given that 2% of the flight times are longer than 82 minutes, find the value of  $\sigma$ . [3]
- (b) Find the probability that a randomly selected flight will have a flight time of more than 80 minutes. [2]
- (c) Given that a flight between the two cities takes longer than 80 minutes, find the probability that it takes less than 82 minutes. [4]

On a particular day, there are 64 flights scheduled between these two cities.

- (d) Find the expected number of flights that will have a flight time of more than 80 minutes. [3]
- (e) Find the probability that more than 6 of the flights on this particular day will have a flight time of more than 80 minutes. [3]



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9. [Maximum mark: 15]

**All answers in this question should be given to four significant figures.**

In a local weekly lottery, tickets cost \$2 each.

In the first week of the lottery, a player will receive  $\$D$  for each ticket, with the probability distribution shown in the following table. For example, the probability of a player receiving \$10 is 0.03. The grand prize in the first week of the lottery is \$1000.

$d$	0	2	10	50	Grand Prize
$P(D = d)$	0.85	$c$	0.03	0.002	0.0001

(a) Find the value of  $c$ . [2]

(b) Determine whether this lottery is a fair game in the first week. Justify your answer. [4]

If nobody wins the grand prize in the first week, the probabilities will remain the same, but the value of the grand prize will be \$2000 in the second week, and the value of the grand prize will continue to double each week until it is won. All other prize amounts will remain the same.

(c) Given that the grand prize is not won and the grand prize continues to double, write an expression in terms of  $n$  for the value of the grand prize in the  $n$ th week of the lottery. [2]

The  $w$ th week is the first week in which the player is expected to make a profit. Ryan knows that if he buys a lottery ticket in the  $w$ th week, his expected profit is  $\$p$ .

(d) Find the value of  $p$ . [7]

**References:**

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12EP11

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12EP12